Exploring dynamical QED effects with the reweighting method

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Trivial?

$$1 + 1 = 2$$

$$100 = 99 + 1$$

Yes, trivial!

Trivial?

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} = 1$$

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-c)^2} = 1$$

Gaussian integral
or Euler-Poisson integral

A little bit complicated.
But yes, mathematically trivial!

Trivial?

Exercise

Calculate a gaussian integral

$$I = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-c)^2} \quad (= 1)$$

numerically by Monte Carlo method (in warped way).

<u>Warped way (but mathematically identical)</u>

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-c)^2} = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} e^{-\frac{c^2}{2}} e^{cx}$$

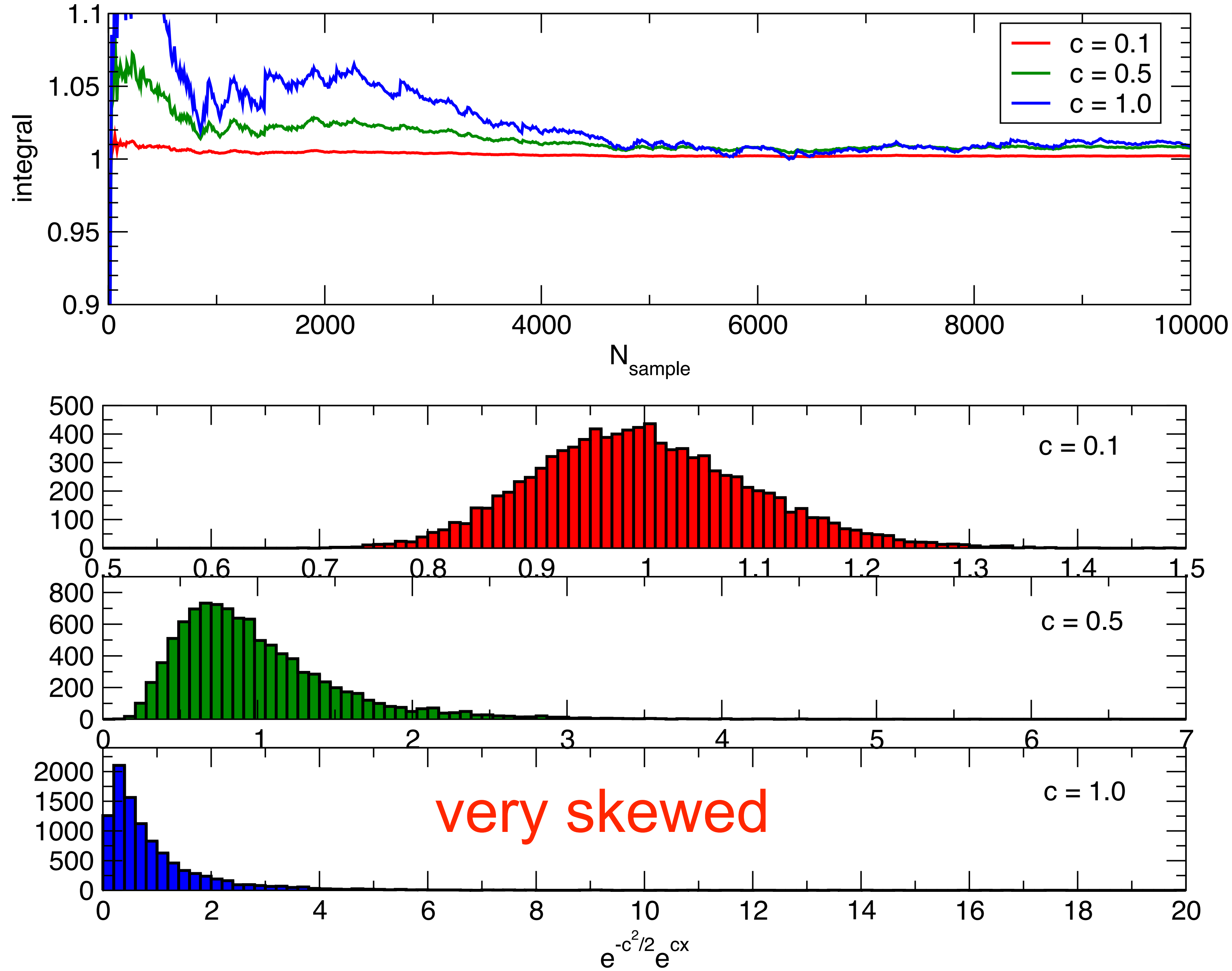
- Generate random numbers with normal gaussian distribution

$$P(x_i) = \frac{e^{-\frac{x_i^2}{2}}}{\sqrt{2\pi}}$$

- Calculate an average $\left\langle e^{-\frac{c^2}{2}} e^{cx} \right\rangle_{x_i} = \frac{1}{N_{\mathrm{sample}}} \sum_i e^{-\frac{c^2}{2}} e^{cx}$

Trivial?

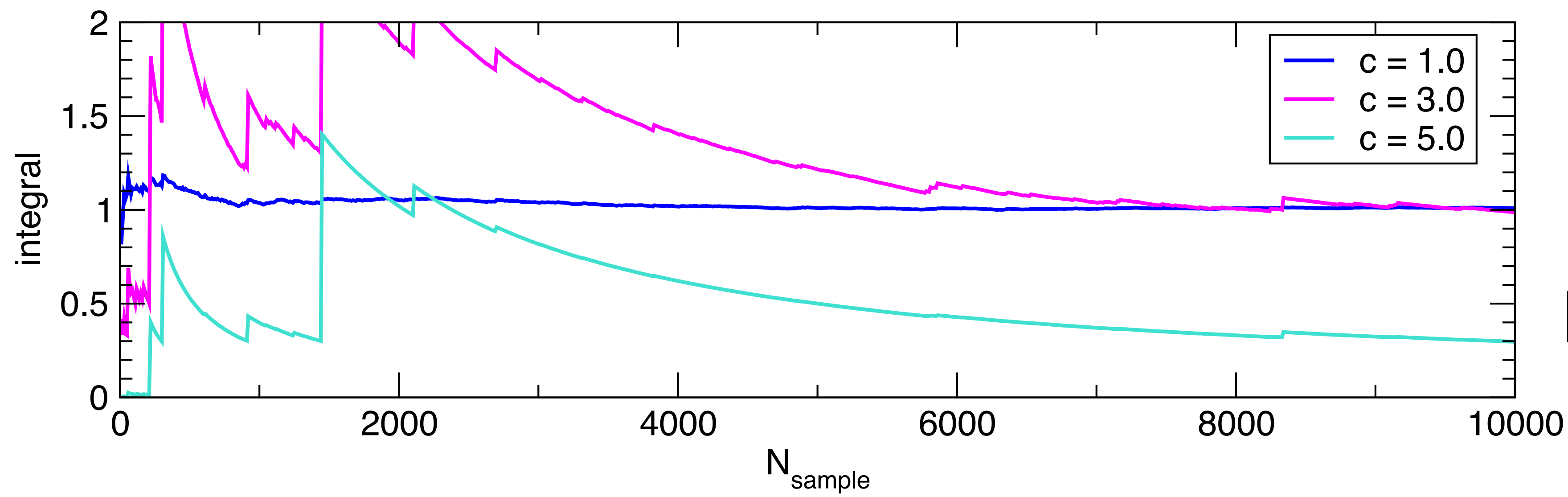
$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-c)^2} = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} e^{-\frac{c^2}{2}} e^{cx}$$



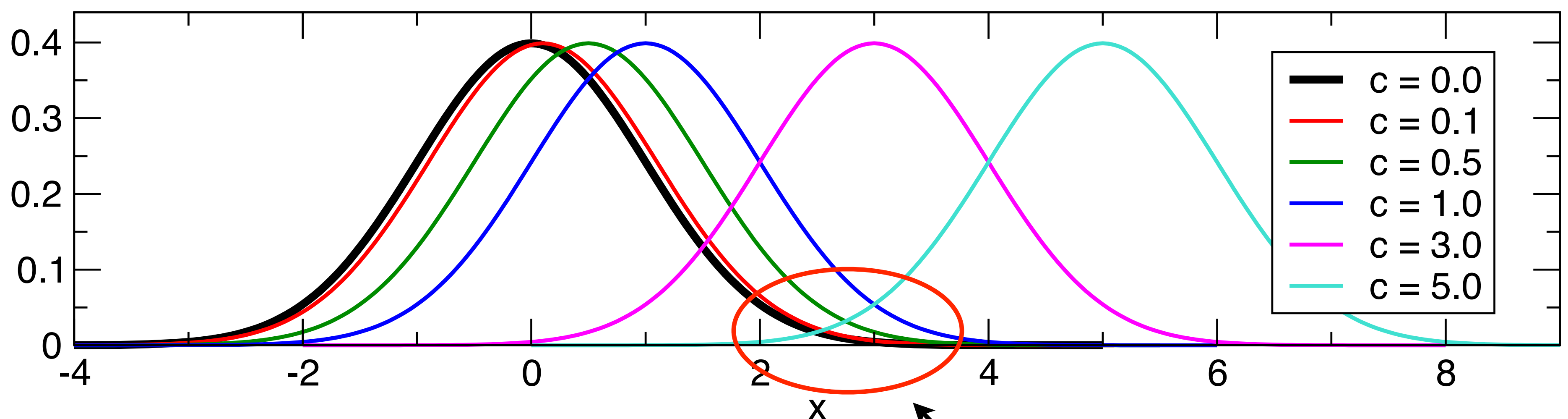
As c goes to large value, the convergence becomes slow.

Trivial?

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-c)^2} = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} e^{-\frac{c^2}{2}} e^{cx}$$



hopeless



Very small overlap between c=0 and 5.

Overlap Problem

Riweighting

- Exercise case

$$\int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{1}{2}(x-c)^2} f(x) = \int_{-\infty}^{\infty} \frac{dx}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} f(x) \boxed{e^{-\frac{c^2}{2}} e^{cx}}$$

reweighting factor to shift the parameter c
in the gaussian distribution

- Field theory case

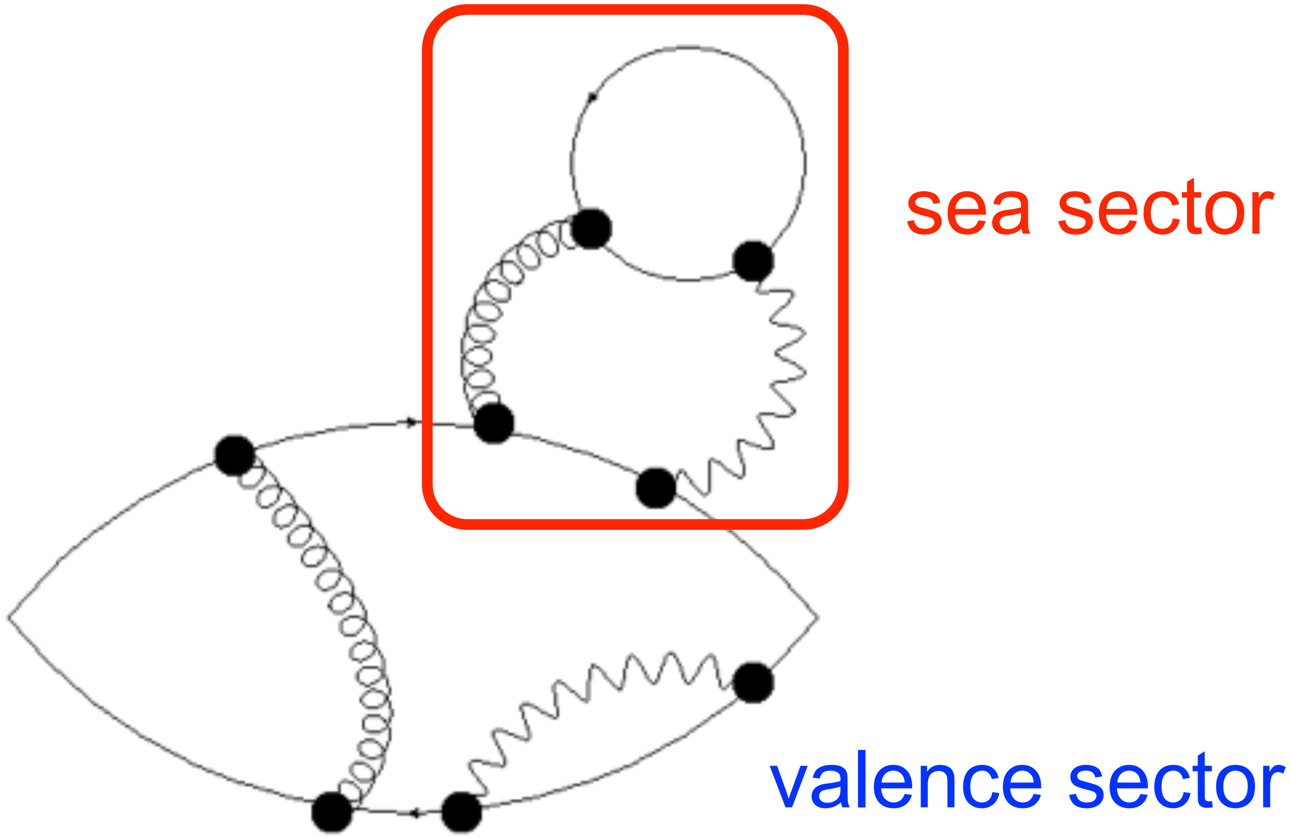
$$\langle f \rangle' = \frac{1}{Z'} \int \mathcal{D}\phi e^{-S'[\phi]} f[\phi] = \frac{\int \mathcal{D}\phi e^{-S[\phi]} f[\phi] w[\phi]}{Z \int \mathcal{D}\phi e^{-S[\phi]} w[\phi]}$$

$$w[\phi] = e^{-(S'[\phi] - S[\phi])}$$

reweighting factor to change the action

What we want to do

- Full QED effect



<u>Add full QED effect by reweighting</u>

Full QCD ensemble	reweighting →	Full QCD+QED calulation

QED reweighting

Full QED from full QED

- Full QCD + full QED

$$\langle O\rangle_{\rm QCD+QED}$$

$$= \frac{\int \mathcal{D}U\mathcal{D}A\mathcal{D}\bar\psi\mathcal{D}\psi O[\widetilde{U},\bar\psi,\psi]e^{-S_f[\bar\psi,\psi,\widetilde{U}]-S_{SU(3)}[U]-S_{U(1)}[A]}}{\int \mathcal{D}U\mathcal{D}A\mathcal{D}\bar\psi\mathcal{D}\psi e^{-S_f[\bar\psi,\psi,\widetilde{U}]-S_{SU(3)}[U]-S_{U(1)}[A]}}$$

$$= \frac{\int \mathcal{D}U\mathcal{D}A O'[\widetilde{U}]e^{\ln\det D[\widetilde{U}]-S_{SU(3)}[U]-S_{U(1)}[A]}}{\int \mathcal{D}U\mathcal{D}A e^{\ln\det D[\widetilde{U}]-S_{SU(3)}[U]-S_{U(1)}[A]}}$$

$$\widetilde{U} = U \times e^{iqeA}$$

Usually, gauge configs are generated w/o A .

- Full QCD + quenched QED   e.g. [Blum et.al. (2007,2010)]

$$\langle O\rangle_{\rm QCD+qQED} = \frac{\int \mathcal{D}U\mathcal{D}A O'[\widetilde{U}]e^{\ln\det D[U]-S_{SU(3)}[U]-S_{U(1)}[A]}}{\int \mathcal{D}U\mathcal{D}A e^{\ln\det D[U]-S_{SU(3)}[U]-S_{U(1)}[A]}}$$

QED reweighting

- Full QED from quenched QED [Duncan et.al.(2005)]
  - Reweight from quenched QED to full QED

$$
\begin{aligned}
&\langle O\rangle_{\mathrm{QCD+QED}} \\
&= \frac{\int \mathcal{D}U\mathcal{D}A O'[\widetilde{U}] e^{\ln\det D[\widetilde{U}] - S_{SU(3)}[U] - S_{U(1)}[A]}}{\int \mathcal{D}U\mathcal{D}A e^{\ln\det D[\widetilde{U}] - S_{SU(3)}[U] - S_{U(1)}[A]}} \\
&= \frac{\int \mathcal{D}U\mathcal{D}A O'[\widetilde{U}] \frac{\det D[\widetilde{U}]}{\det D[U]} e^{\ln\det D[U] - S_{SU(3)}[U] - S_{U(1)}[A]}}{\int \mathcal{D}U\mathcal{D}A \frac{\det D[\widetilde{U}]}{\det D[U]} e^{\ln\det D[U] - S_{SU(3)}[U] - S_{U(1)}[A]}}
\end{aligned}
$$

Full QED effects are taken into account by the reweighting factor:

$$
w[U_{\mathrm{QCD}}, A] = \frac{\det D[U_{\mathrm{QCD}} \times e^{iqeA}]}{\det D[U_{\mathrm{QCD}}]}
$$

on the dynamical QCD configuration U_{QCD} .

Perturbative picture of QED

$$\alpha = \frac{e^2}{4\pi} \sim \frac{1}{137}$$

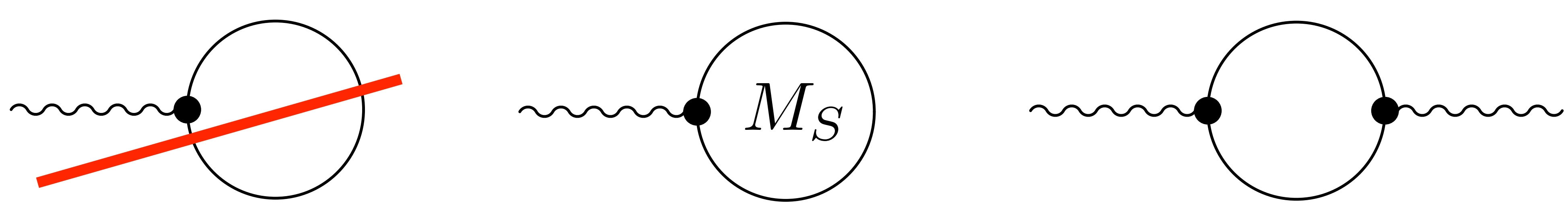
- Reweighting factor in Nf=3

$$\begin{aligned} w &= 1 + \cancel{w_1 \mathrm{tr}(Q_S)}e + w_{1m}\mathrm{tr}(M_S Q_S)e + w_2 \mathrm{tr} Q_S^2 e^2 \\ &\quad + \mathcal{O}(M_S^2 e, M_S e^2, e^3) \end{aligned}$$

$$Q_S = \mathrm{diag}(q_u, q_d, q_s) = \mathrm{diag}\left(+\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3}\right)$$

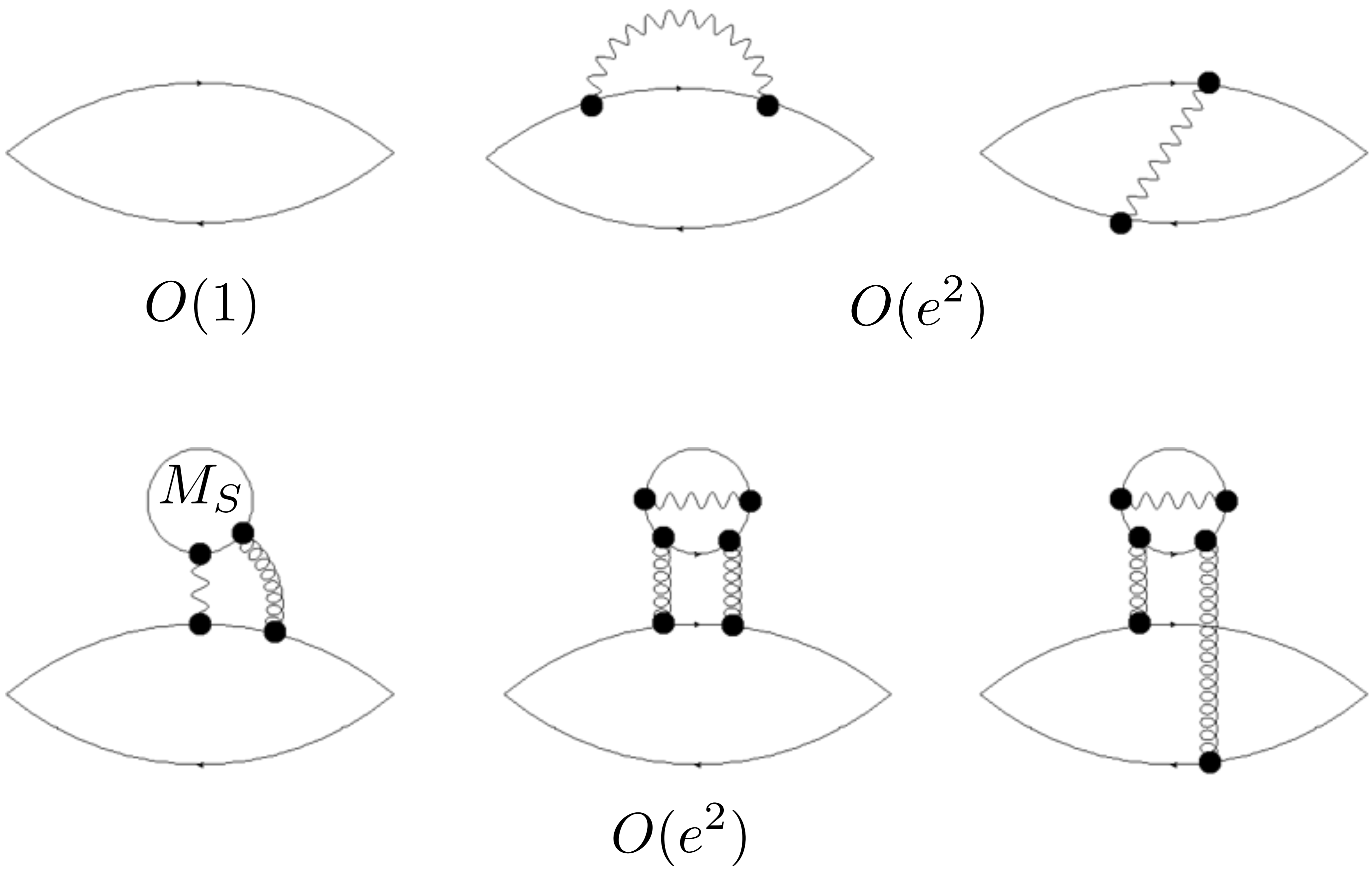
$$M_S = \mathrm{diag}(m_u, m_d, m_s)$$

$$\mathrm{tr}(Q_S) = +\frac{2}{3} - \frac{1}{3} - \frac{1}{3} = 0$$



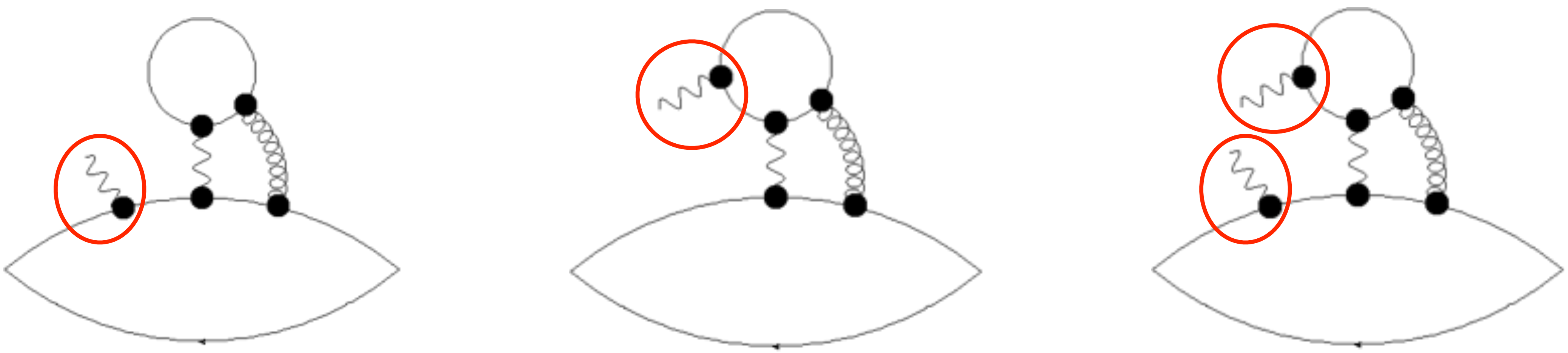
Perturbative picture of QED

- Coupled with sea sector



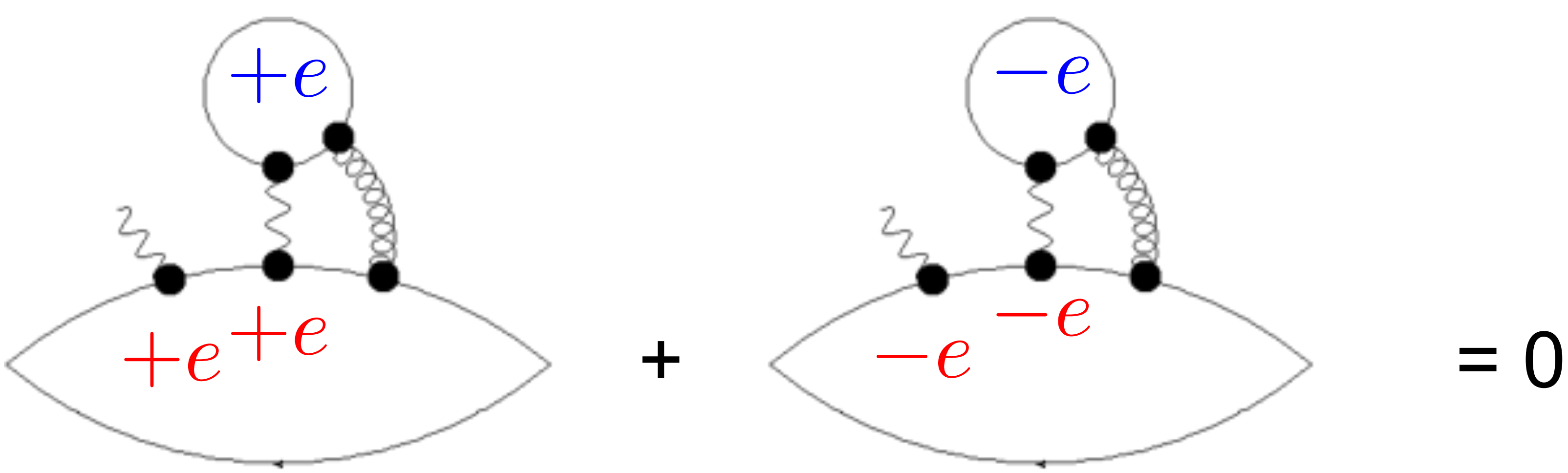
Reduction of unphysical noise

- Unphysical contribution



At finite statistics, unphysical contributions could be remained.
They could cause large noise in the correlator.

- +e/-e trick [Blum et.al. (2007)]



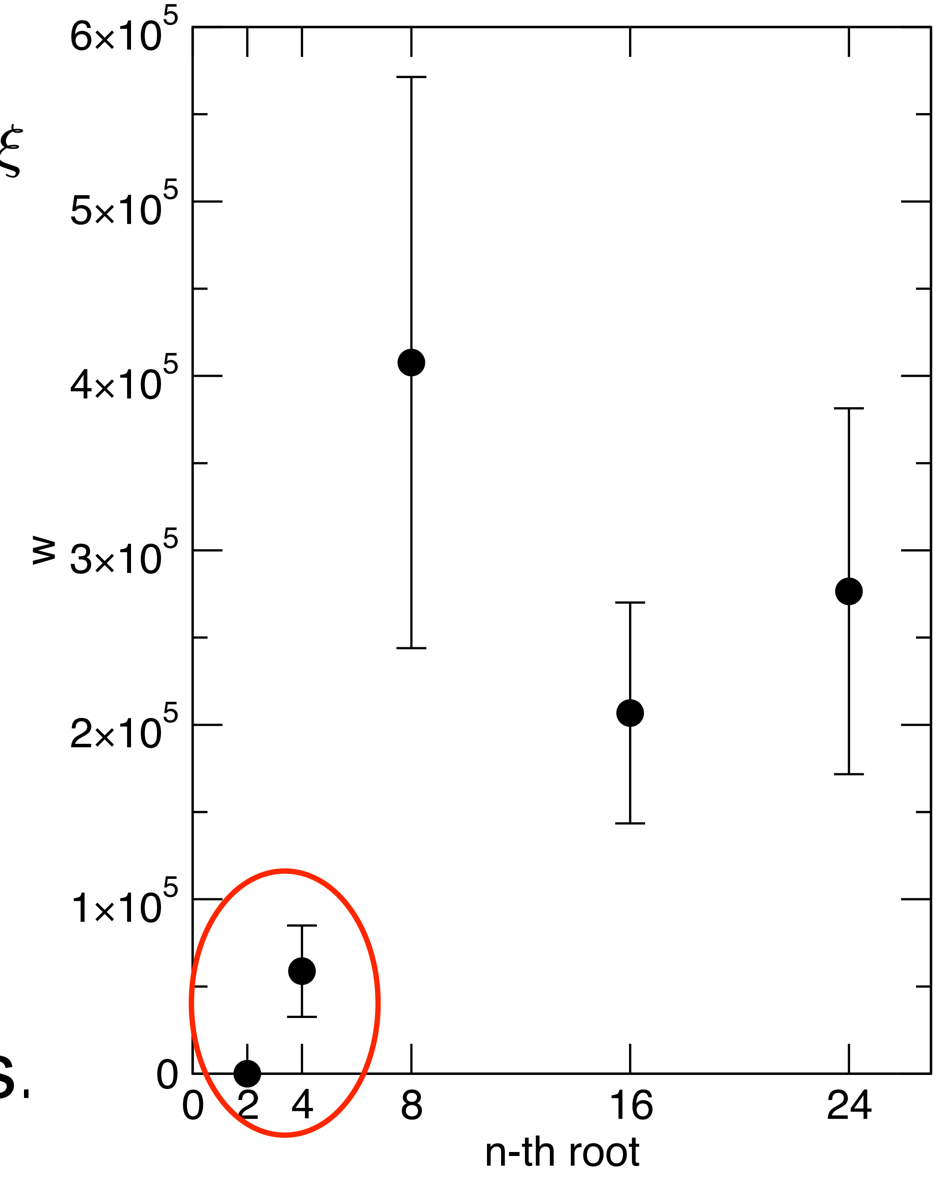
At least, e odd contributions are exactly removed.

Calculation of reweight factor

$$w = \frac{\det D[\tilde{U}]}{\det D[U]}$$

- Stochastic estimation
  - Random gaussian vectors ξ are used.

$$\det \Omega = \frac{\int \mathcal{D}\xi e^{-\xi^\dagger \Omega^{-1} \xi}}{\int \mathcal{D}\xi e^{-\xi^\dagger \xi}} = \langle e^{-\xi^\dagger (\Omega^{-1} - 1)\xi} \rangle_\xi$$



- Root trick
  - Usually, exponents largely fluctuate. The distribution of the reweight factor is largely skewed.
  - Use mathematically identical relation to reduce the contributions from the outliers.

$$\det \Omega = (\det \Omega^{1/n})^n = \prod_{i=1}^{n} \langle e^{-\xi_i^\dagger (\Omega^{-1/n} - 1)\xi_i} \rangle_{\xi_i}$$

Simulation Parameters

- Nf=2+1 dynamical domain-wall fermion + Iwasaki gluon configurations [RBC+UKQCD]
  - $\beta = 2.15 \ (a^{-1} = 1.78 \text{ GeV}), L^3 \times T = 16^3 \times 32 \ ((1.8 \text{ fm})^3)$
  - $[m_{ud}, m_s] = [0.01, 0.04] \ (m_\pi \sim 450 \text{ MeV})$
  - 3500 trajectories, measured on every 20 trajectories, bin size = 60 trajectories.
- Non-compact U(1) gauge configs

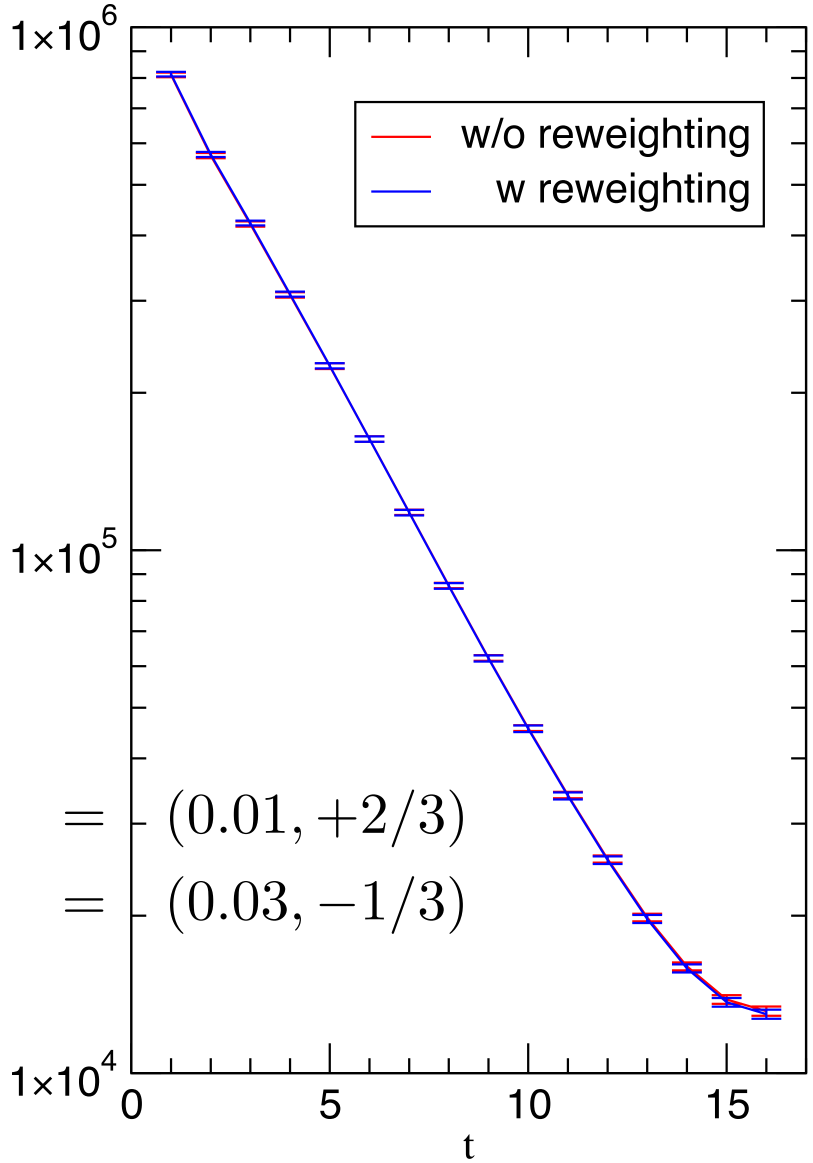
$$S_{U(1)} = \frac{1}{4e^2} \sum (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$$

  - Generated in the quenched QED study [Blum et.al. (2010)]
- Calculation of reweighting factor
  - 24-th root trick is used.
  - Maximally 384 random gaussian noise vectors are used for each reweighting factor.

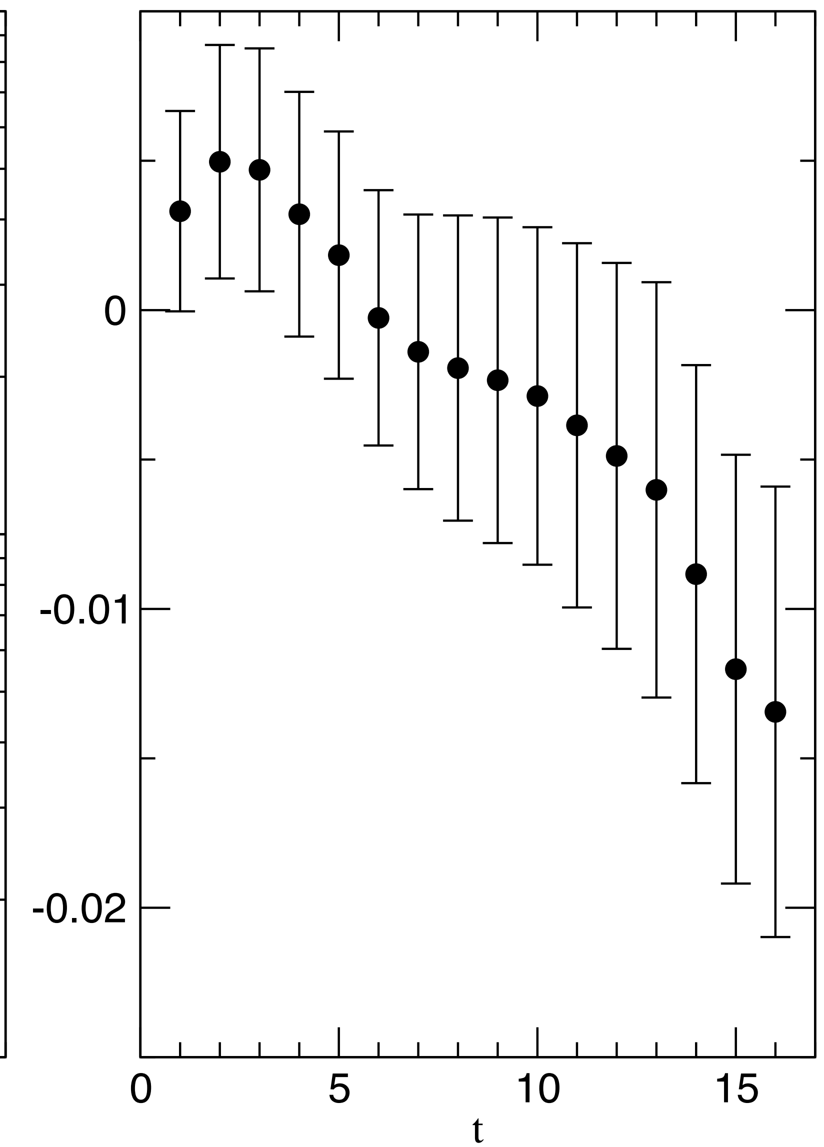
Some results

- Full QED effect on PS meson correlator

$$C(t) = \langle P(t)P(0) \rangle \qquad \frac{C(t)[e_S = e_{phys}] - C(t)[e_S = 0]}{C(t)[e_S = 0]}$$



$$(m_{v_1}, q_{v_1}) = (0.01, +2/3)$$
$$(m_{v_2}, q_{v_2}) = (0.03, -1/3)$$

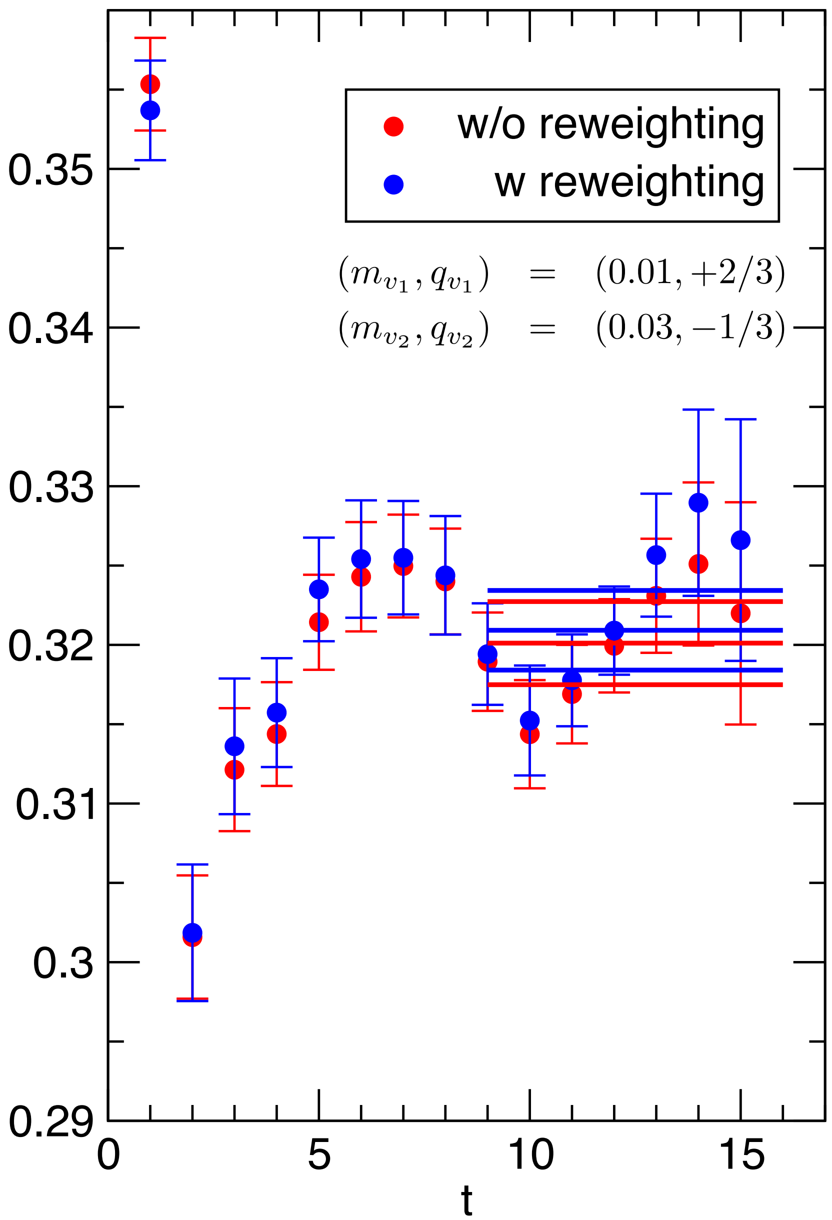


faint effect

Some results

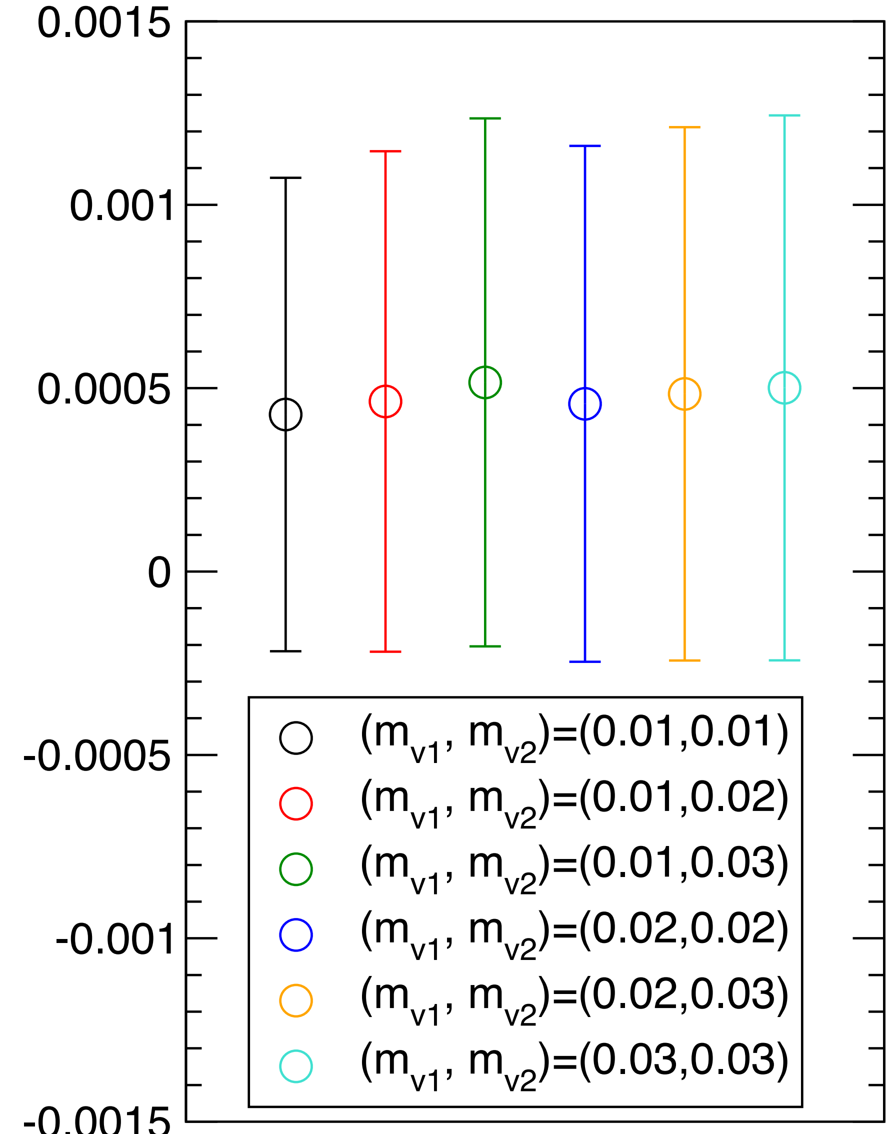
- PS meson mass

$$m_{\rm PS}^{eff} = -\ln(C(t+1)/C(t))$$



Full QED effect

$$m_{\rm PS}^2[e_S = e_{phys}] - m_{\rm PS}^2[e_S = 0]$$



$(q_{v1}, q_{v2}) = (+2/3, -1/3)$

Error is large. Quark mass dependence is not clear.

Checking the validity

Full QED effect in ChPT

- SU(3), NLO, partially quenched [Bijnens and Danielsson (2007)]

e_S : sea

e_V : valence

$$
\begin{aligned}
&\Delta M^2_{PS(v_1 v_2)} \\
&= M^2_{PS(v_1 v_2)}(e_S \neq 0) - M^2_{PS(v_1 v_2)}(e_S = 0) \\
&= e_S e_V \frac{C}{F_0^4} \frac{1}{8\pi^2} \left\{ \left(\chi_{v_1 u} \ln \frac{\chi_{v_1 u}}{\mu^2} q_u + \chi_{v_1 d} \ln \frac{\chi_{v_1 d}}{\mu^2} q_d + \chi_{v_1 s} \ln \frac{\chi_{v_1 s}}{\mu^2} q_s \right) \right. \\
&\quad \left. - \left(\chi_{v_2 u} \ln \frac{\chi_{v_2 u}}{\mu^2} q_u + \chi_{v_2 d} \ln \frac{\chi_{v_2 d}}{\mu^2} q_d + \chi_{v_2 s} \ln \frac{\chi_{v_2 s}}{\mu^2} q_s \right) \right\} (q_{v_1} - q_{v_2}) \\
&\quad -12 e_S^2 Y_1 \bar{q}^2 \chi_{v_1 v_2} + O(e_S e_V^3, e_S^2 e_V^2, e_S^3 e_V, e_S^4).
\end{aligned}
$$

$$
\chi_{ij} = B_0(m_i + m_j),\ \bar{q}^2 = \frac{1}{3}(q_u^2 + q_d^2 + q_s^2)
$$

$C \longleftarrow$ LO e_V^2 term

Y_1 : New, e_S^2 term

$10^7 C = 2.2(2.0)$
(quenched QED)
[Blum et.al. (2010)]

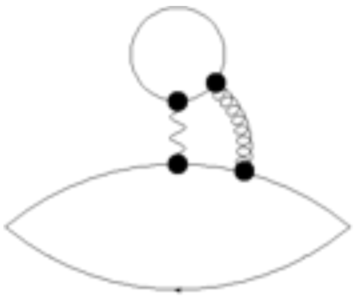
It's hard to see the obtained data obeys this.

Checking the validity

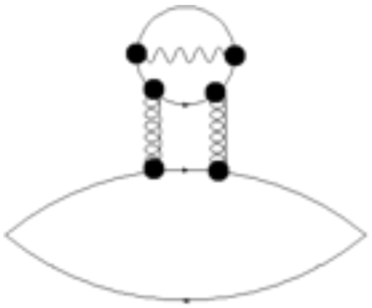
- Separation of terms : +e/-e trick again
  - We can set EM charges in valence and sea sector separately.

$$\begin{array}{rcl} & & (e_S, e_V) \ \ (e_S, e_V) \\ \mathcal{P} & = & (+,+) + (-,-) \\ \mathcal{M} & = & (+,-) + (-,+) \end{array}$$

$$\mathcal{P} - \mathcal{M} \longrightarrow e_S \text{ odd terms}$$



$$\mathcal{P} + \mathcal{M} \longrightarrow e_S \text{ even terms}$$



Checking the validity

- ChPT formula tells us :

$$
\begin{aligned}
&\Delta M^2_{PS(v_1 v_2)} \\
&= M^2_{PS(v_1 v_2)}(e_S \neq 0) - M^2_{PS(v_1 v_2)}(e_S = 0) \\
&= e_S e_V \frac{C}{F_0^4} \frac{1}{8\pi^2} \left\{ \left(\chi_{v_1 u} \ln \frac{\chi_{v_1 u}}{\mu^2} q_u + \chi_{v_1 d} \ln \frac{\chi_{v_1 d}}{\mu^2} q_d + \chi_{v_1 s} \ln \frac{\chi_{v_1 s}}{\mu^2} q_s \right) \right. \\
&\quad \left. - \left(\chi_{v_2 u} \ln \frac{\chi_{v_2 u}}{\mu^2} q_u + \chi_{v_2 d} \ln \frac{\chi_{v_2 d}}{\mu^2} q_d + \chi_{v_2 s} \ln \frac{\chi_{v_2 s}}{\mu^2} q_s \right) \right\} (q_{v_1} - q_{v_2}) \\
&\quad -12 e_S^2 Y_1 \bar{q}^2 \chi_{v_1 v_2} + O(e_S e_V^3, e_S^2 e_V^2, e_S^3 e_V, e_S^4).
\end{aligned}
$$

$$
\chi_{ij} = B_0(m_i + m_j), \;\; \bar{q}^2 = \frac{1}{3}(q_u^2 + q_d^2 + q_s^2)
$$

- Invariant under

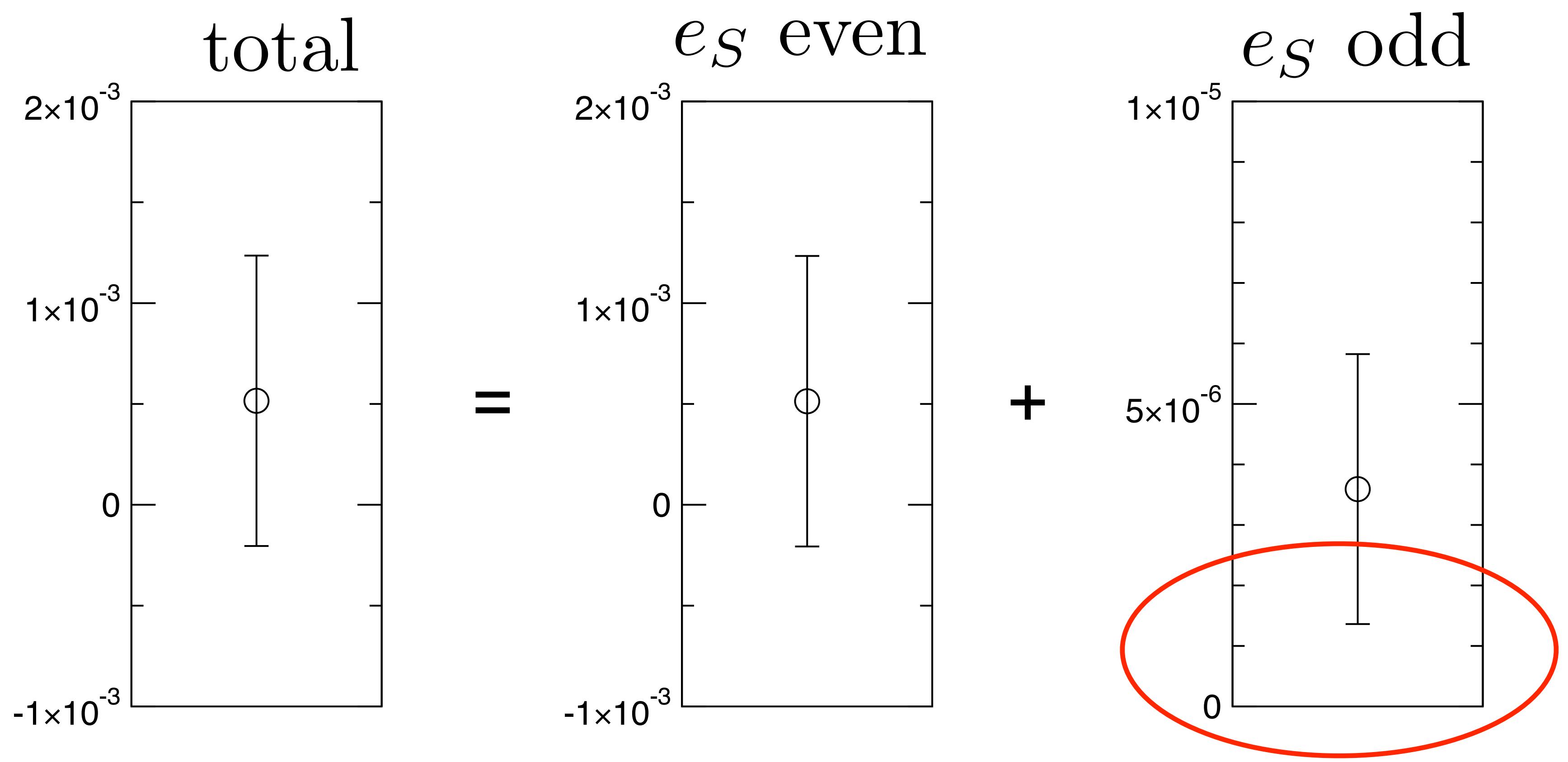
$$
\boxed{\begin{matrix}(m_{v_1}, m_{v_2}) \\ (q_{v_1}, q_{v_2})\end{matrix} \Leftrightarrow \begin{matrix}(m_{v_2}, m_{v_1}) \\ (q_{v_2}, q_{v_1})\end{matrix}}
\qquad
\boxed{\begin{matrix}(m_{v_1}, m_{v_2}) \\ (q_{v_1}, q_{v_2})\end{matrix} \Leftrightarrow \begin{matrix}(m_{v_2}, m_{v_1}) \\ (-q_{v_1}, -q_{v_2})\end{matrix}}
$$

usual

tent to be anti-correlated (?)
The fluctuation could be reduced.

Checking the validity

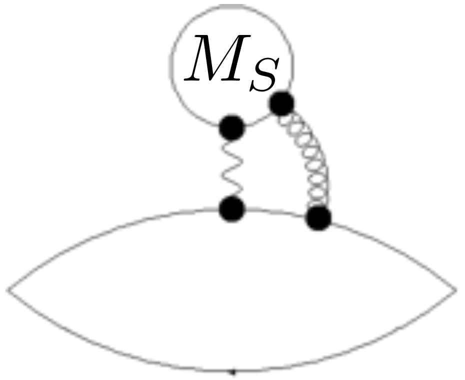
- After the separation :



not consistent with zero.

- e_S even terms $\gg e_S$ odd terms
- $e_S e_V$ term could be suppressed by

$$\frac{m_{v1} - m_{v2}}{\Lambda_{\mathrm{QCD}}} \cdot \frac{\mathrm{tr} M_S Q}{\Lambda_{\mathrm{QCD}}}$$

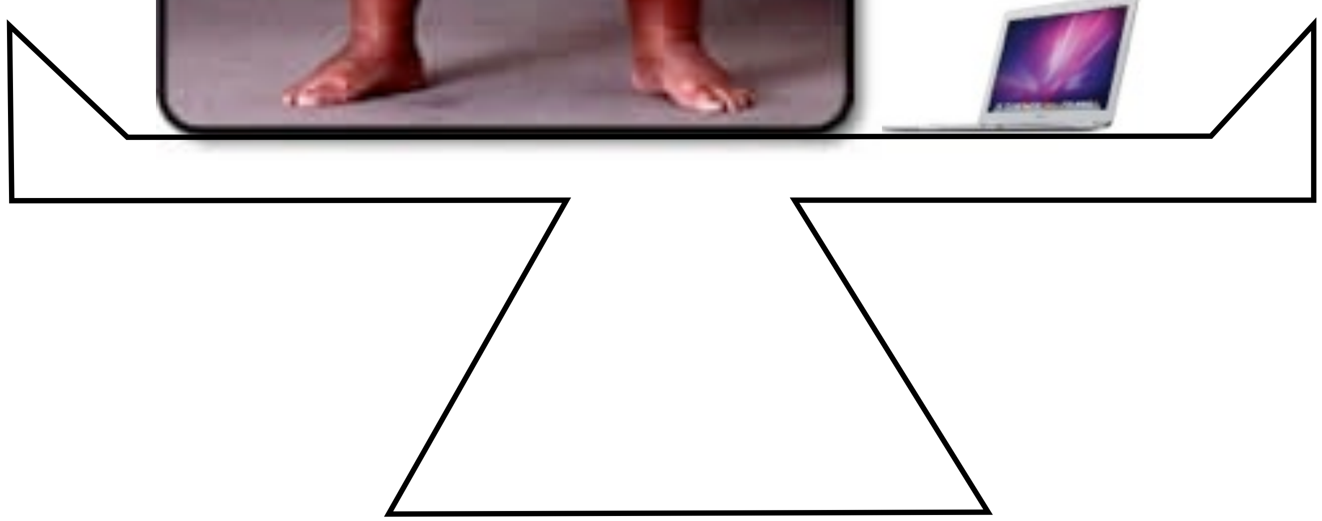


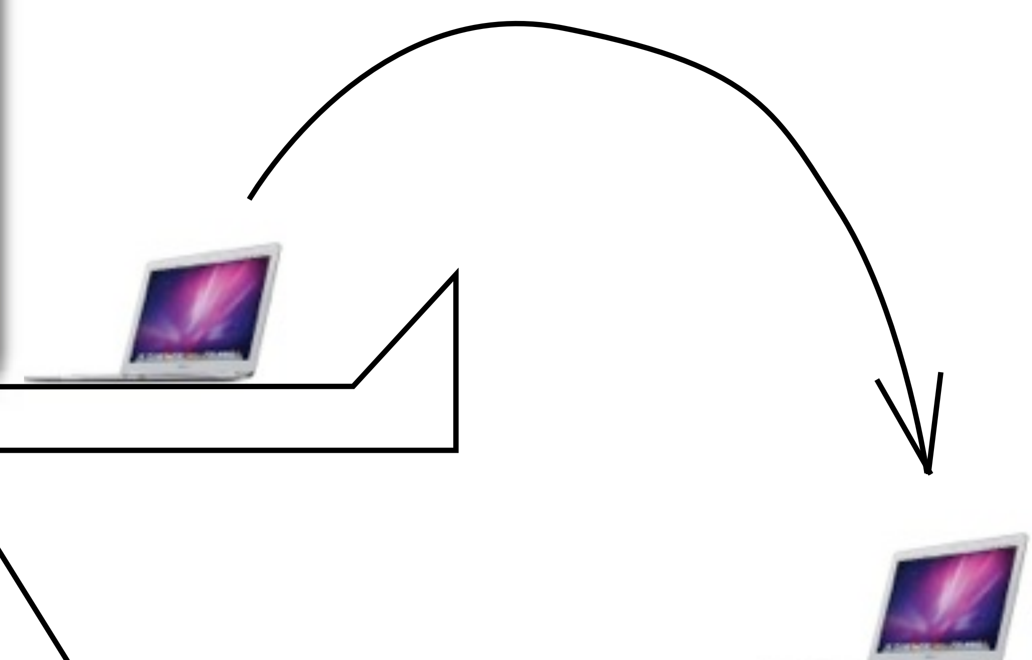
Checking the validity

~ 250Kg

KONISHIKI **小錦**

Sumo Wrestler
(highest rank: Ozeki)



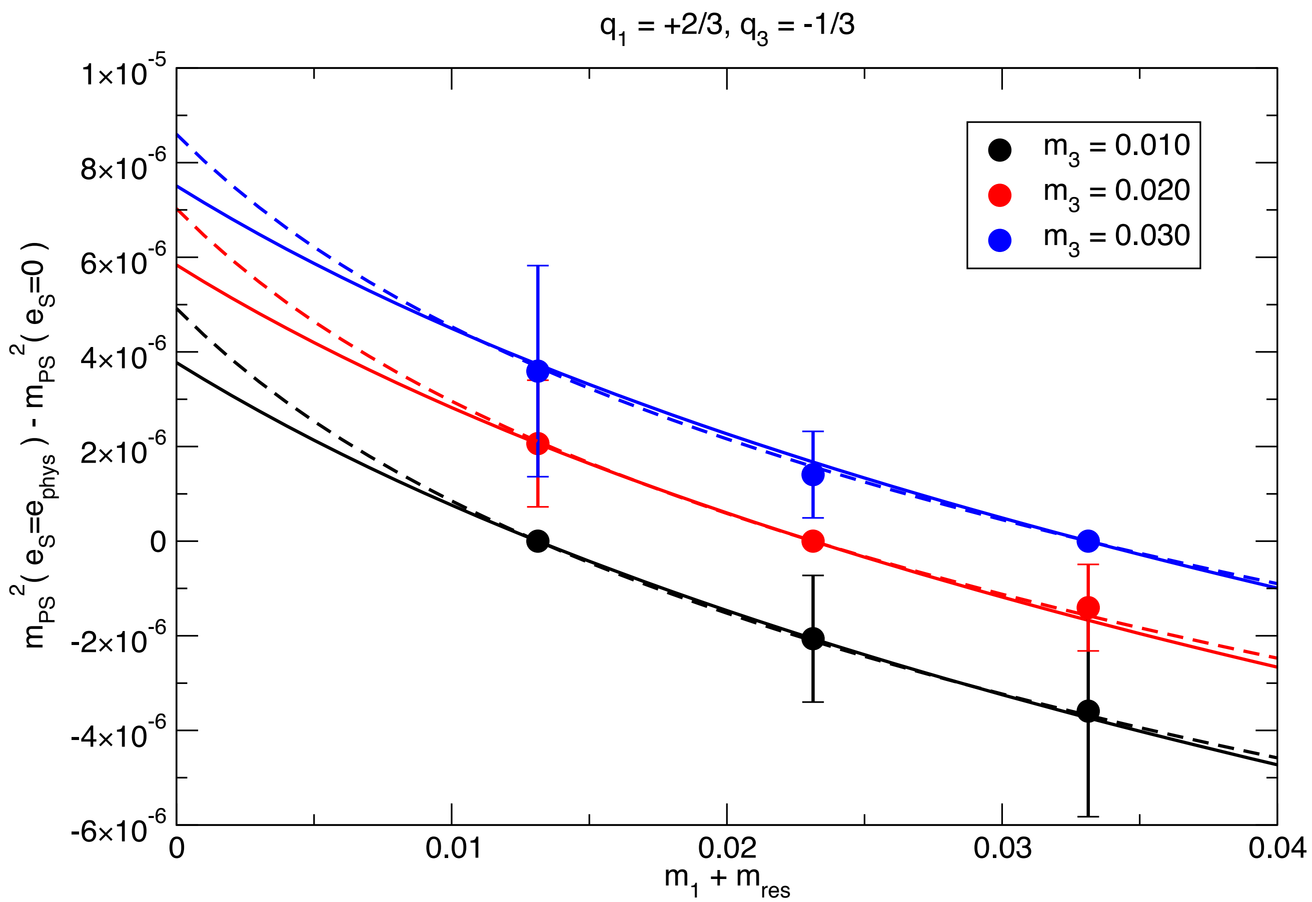


~ 1Kg

Checking the validity

- ChPT fit

$e_S e_V$ term only



from reweighting

$$10^7 C = \begin{cases} 5.1(3.3) & \text{(inf. vol.)} \\ 4.3(2.8) & \text{(fin. vol.)} \end{cases}$$

[this work]

from quenched QED

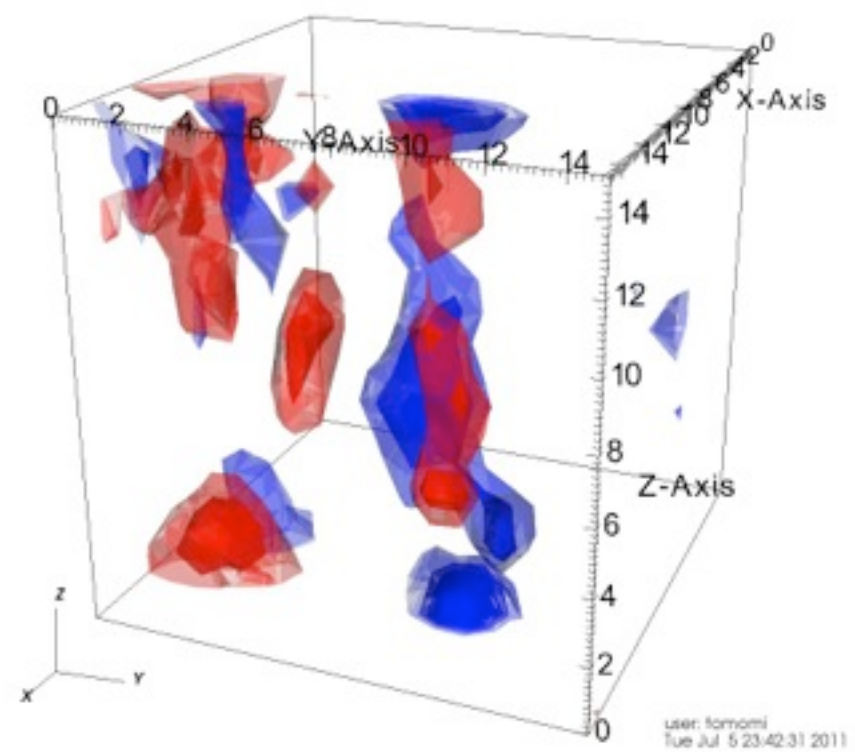
$$10^7 C = \begin{cases} 2.2(2.0) & \text{(inf. vol.)} \\ 9.3(2.4) & \text{(fin. vol.)} \end{cases}$$

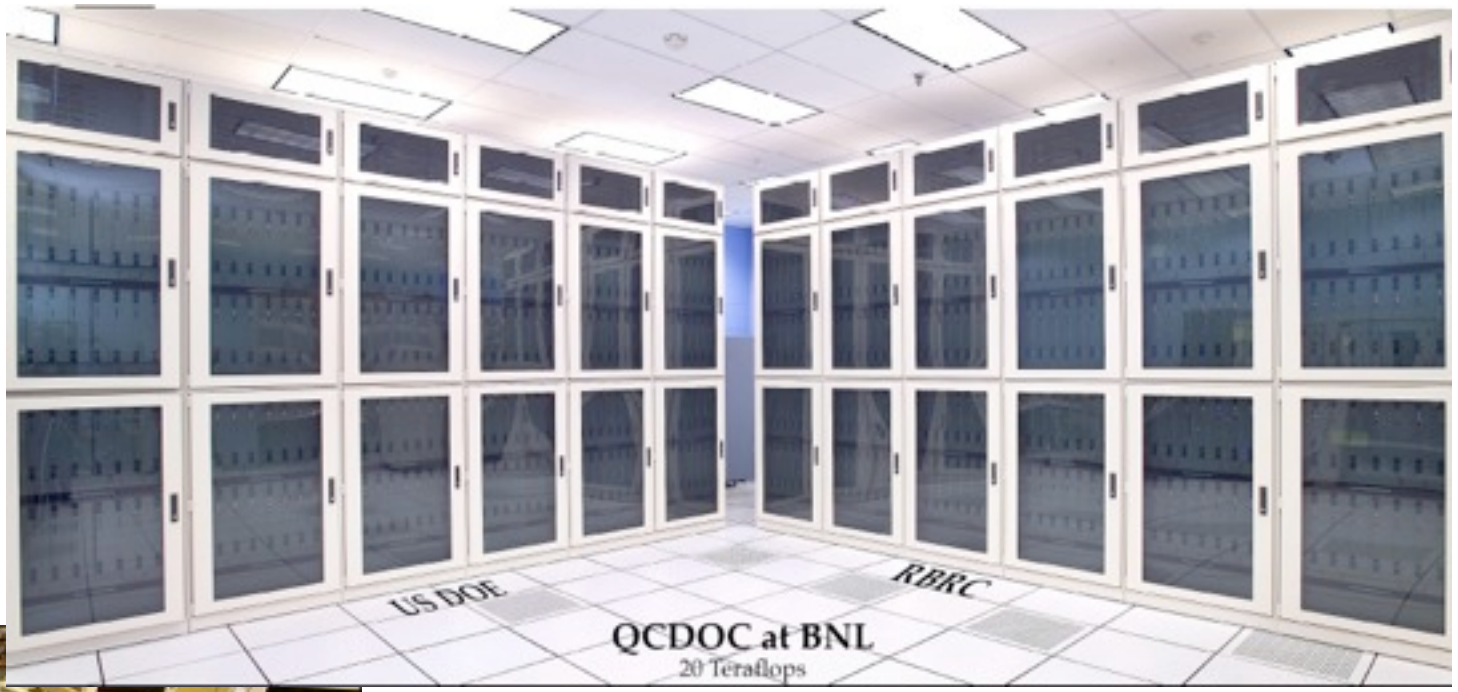
[Blum et.al. (2010)]

At least, order of C is consistent.
Reweighting seems to be well controlled.

Summary

- Full QED effects are added by the reweighting method.
  - +e/-e trick is powerful. e_S even and odd terms can be separated.
  - Seeing $e_S e_V$ term, the reweighting seems to be well controlled.
  - For e_S^2 term, further improvement is needed. Low-mode averaging(?)
- Applications
  - Spectrum with EM
  - $g_\mu - 2$
  - Chiral Magnetic Effect





Thank you!